A Mathematical Model for use in Diabetes Control

Michael Hill *BSc(Natal)*
Computer Centre
Natal Technikon
Berea Road
Durban

October 1986

1 Introduction

It seems possible that the problem of blood sugar level control in Type I Diabetes could be aided by the use of a mathematical model of the situation. This could then be used in conjunction with a computer in order to obtain more accurate and dynamic control of the blood sugar level.

What is required is the construction of a mathematical model which describes quantitatively the factors which effect the blood sugar level in a diabetic.

If such a system could be developed it, could then be used on a computer in order to calculate changes in insulin dose, food intake and exercise periods to achieve the best possible control over a period of time.

Such a mathematical model may also be useful in the general study of control in diabetes since it could be used to give a more accurate quantititative idea of the situation.

The variables involved are:
i) Blood sugar level
ii) Insulin intake
iii) Food intake
iv) Exercise
v) Other factors, eg. insulin opposing hormones, stress, illness, and such like.

i) and ii) can be very accurately measured, iii) fairly accurately and it should be possible to calculate a reasonable estimate for iv). If there are factors involved in v) which can not be ignored then their effects will have to be investigated and taken into account.

It is then necessary to develop a mathematical model of the situation giving the quantititative effects of ii), iii) and iv) on i).

The basic equation should be of the form:

$$G(t) = \int_0^t G(0) + F(t) - I(t) - E(t)dt$$

where

G(t) is blood sugar level as a function of time
G(0) is the intial blood sugar level
F(t) is the effect of food intake over time on G
I(t) is the effect of insulin over time on G
E(t) is the effect of exercise over time on G

Various constants involved in the equations will be dependent on the individual concerned, since the effect of a given amount of insulin, intake of food, etc. on the level of a person's blood sugar will depend on the physical attributes of the person. The validity of the model would depend on these values remaining fairly constant.

There are two phases involved to implement such a system:

2 Phase I

The first thing to do is to determine whether it is possible to set up the system so that given the values of initial blood sugar level, food intake, insulin intake and activity levels, it can extrapolate the blood sugar level over a period of time.

The functions for F(t), I(t) and E(t) represented by polynomials. (They can be approximated by interpolative polynomials.)

Let us assume that interpolative polynomials of order 2 are sufficient.

Thus:

$$F_n(t) = a_{n;2}t^2 + a_{n;1}t + a_{n;0} \quad 0 < t < t_n$$

$$E(t) = -k(b_2t^2 + b_1t + b_0) \quad 0 < t < t_b$$

$$I_i(t,k) = -(c(k)_{i;2}t^2 + c(k)_{i;1}t + c(k)_{i;0}) \quad 0 < t < t_i$$

where:

F_n is the function for the n^{th} food item
I_i is the function for the i^{th} type of insulin (fast, medium of slow acting)
k is the level of exercise
$I(t,k)$ includes the exercise level factor (k) since it is known that insulin absorption rate is affected by physical activity due to increased blood flow during periods of exercise.

In order for the model to be useful the error levels of the calculations will have to be small enough to keep the calculated values within acceptable ranges.

Keeping blood sugar values between 100 and 200 mg/dl is considered to be reasonable control. Thus, for example, (don't take these figures too seriously!) if a value of 130 mg/dl is aimed for, and the error bound is 25 mg/dl, then values in the range 105 - 155 mg/dl can be expected to be maintained. The point here is that although extrapolated values can by no means be expected to be exact, the bounds for the error levels do not have to be all that small for the model to be useful.

It should also be possible to deduce from such a model the periods during the day/night during which the blood sugar levels are likely to deviate from levels which are considered to be acceptable.

3 Phase II

If Phase I can be successfully completed, the next step is to develop a program to implement the system on a computer. With so many variables involved, in order to have a manageable situation most of the variables will have to be held constant and the best values for the others calculated.

A reasonable way may be to keep F and E at predetermined values, and then calculate the best doses of insulin to produce an I curve which will give optimal control. Along with proposed values for F and E over the period concerned, restrictions for the calculations of doses could be input, such as the periods during which they could be given, and the maximum permissable number of injections allowed within a period, thus further limiting the number of free variables which need to be calculated, and giving the users a reasonable amount of control over the recommended doses of insulin.

Once the program has been developed it could then be run on almost any computer (restrictions being the amount of memory required, and perhaps the processing time required).

It is also possible that it could be built into a dedicated machine (hard-wired with the program in ROM). Such a machine should be less expensive than a normal microcomputer. This kind of machine could be something like the present day blood glucose testing machines.